Near-Optimal-Time Quantum Algorithms for Approximate Pattern Matching

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Strings and Similarity Measures Between Strings

- A *string* is a sequence of characters.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
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- The *Hamming distance* $\mathrm{HD}(X, Y)$ counts the number of mismatches between X, Y.

X	b	b	a	a	a	b	b	b	a	b	b	a	a
Y	a	b	a	b	a	b	b	b	a	b	b	a	a

$\mathrm{HD}(X, Y) = 2$

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```
X    b b a a a b b b a b b a a
Y    a b a b a b b b a b b a a
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$\mathrm{HD}(X, Y) = 2$

- The *edit distance* $\mathrm{ED}(X, Y)$ measures the minimum number of insertions, deletions, and substitutions of characters to transform X into Y.

```
X    a b a a b a b a b a b b b a
Y    a b a b a b b b a b b a a
```

$\mathrm{ED}(X, Y) = 3$

Pattern Matching (PM)

Text T, $|T| = n$ a b a a b a b a b a b b b a b b a a a b b b a b b a a

Pattern P, $|P| = m$ a b a b a b b b a b b a a

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- Input strings P/T given as oracle where queries can be made in **superposition**
- **Query complexity** $Q(n)$: counts number of queries to oracle
- **Time complexity** $T(n)$: also counts the number of elementary gates

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We assume $\boldsymbol{n \leq 3/2 \cdot m}$.

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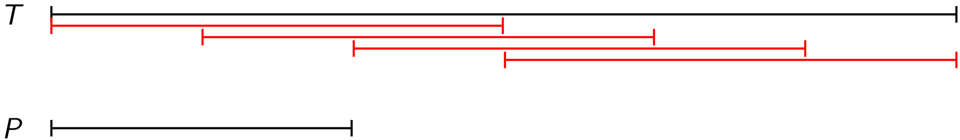

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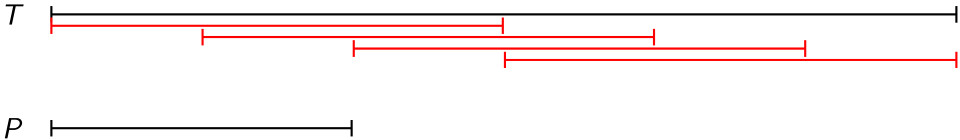

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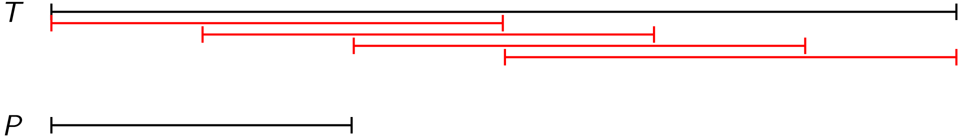

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- To compute $\mathrm{Occ}_k^H(P, T) \neq \emptyset$ (resp. $\mathrm{Occ}_k^H(P, T) \neq \emptyset$) incurs a $\mathcal{O}(\sqrt{n/m})$ overhead.

Our Results

Refererence	Setting	Query Complexity	Time Complexity	Notes
[CKW20]	mismatches	$\widetilde{\mathcal{O}}(k^2\sqrt{n})$	$\widetilde{\mathcal{O}}(k^2\sqrt{n})$	

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- Offer advantage over classical algorithms for $k \leq n^{1/4}$ for mismatches and for $k \leq n^{1/7}$ for edits.

Previous Approach

[CKW20]: Find a candidate set $\mathrm{Occ}_k^H(P,T) \subseteq C$ in one of two forms.

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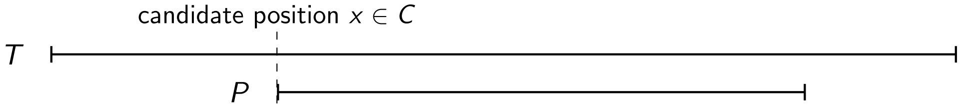

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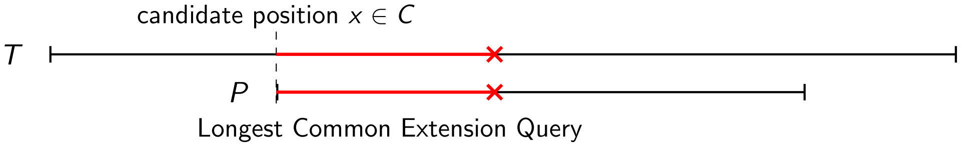

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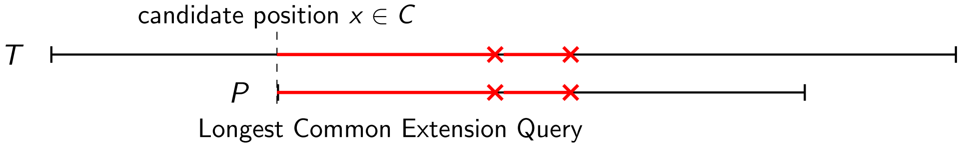

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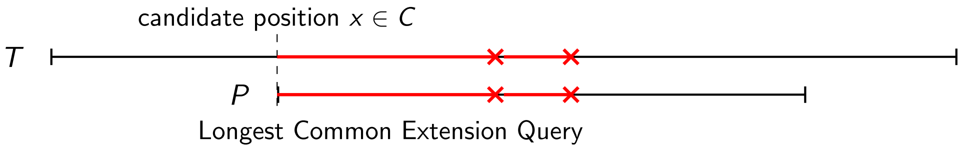

Problem: $\mathcal{O}(k)$ are too many candidate positions!

Workaround: Communication Complexity

[CKP19]: A subset $S \subseteq \mathrm{Occ}_k^H(P, T)$ of size $|S| = \mathcal{O}(\log n)$ suffices to encode $\mathrm{Occ}_k^H(P, T)$.

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Theorem [CKP19]

There exists a subset $S \subseteq \mathrm{Occ}^H_k(P, T)$ of size $|S| = \mathcal{O}(\log n)$ such that the mismatch information for all $x \in S$, defined as

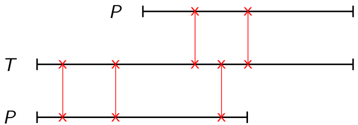
$$\mathrm{MI}(x) := \{(i, P[i], T[x+i]) \mid i \in [0..m) \text{ and } P[i] \neq T[x+i]\},$$

provides enough information to construct two strings $P^{\#}$ and $T^{\#}$ satisfying

$$\mathrm{Occ}^H_k(P, T) = \mathrm{Occ}^H_k(P^{\#}, T^{\#}).$$

Construction of $P^{\#}$ and $T^{\#}$

Select a subset $\{0, n-m\} \subseteq S \subseteq \mathrm{Occ}_k^H(P, T)$.



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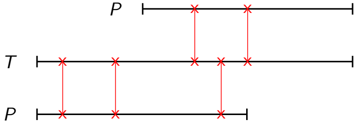

From $\{\mathrm{MI}(x) \mid x \in S\}$ we can infer a set $\mathcal{E}(S)$ of equalities of the form:

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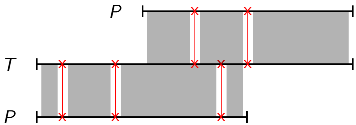

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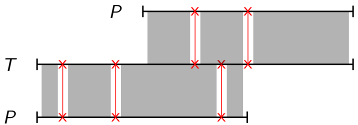

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Definition

$P^{\#}$, $T^{\#}$ are strings of the same length of P, T, and $T^{\#}[i] = \sigma$, $P^{\#}[j] = \sigma'$, and $P^{\#}[j] = T^{\#}[i]$ if and only if such equalities can be inferred from $\mathcal{E}(S)$.

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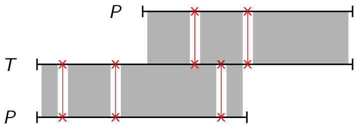

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Theorem [CKP19]

For all $x \in [0..n-m]$ we have $\mathrm{HD}(P^{\#}, T^{\#}[x..x+m)) \geq \mathrm{HD}(P, T[x..x+m))$.
Moreover, if x divides $\gcd(S)$, then equality holds.

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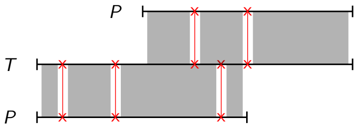

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By choosing S s.t. $\gcd(S) = \gcd(\mathrm{Occ}_k^H(P, T))$ we obtain $P^{\#}, T^{\#}$ s.t. $\mathrm{Occ}_k^H(P, T) = \mathrm{Occ}_k^H(P^{\#}, T^{\#})$.

Constructing S through a Candidate Set

[CKW20]: Find a candidate set $\mathrm{Occ}_k^H(P, T) \subseteq C$ in one of two forms.

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For each $x \in C$, distinguish between $x \in \mathrm{Occ}^H_k(P,T)$ and $x \notin \mathrm{Occ}^H_{5k}(P,T)$

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C forms an arithmetic progression and $C \subseteq \mathrm{Occ}^H_{5k}(P,T)$

The set C satisfies $\mathrm{Occ}^H_k(P,T) \subseteq C \subseteq \mathrm{Occ}^H_{5k}(P,T)$.
Choose $\{0, n-m\} \subseteq S \subseteq C$ s.t. $\gcd(S) = \gcd(C)$ and $|S| = \mathcal{O}(\log n)$.

Construct compressed $P^{\#}$ and $T^{\#}$ and compute $\mathrm{Occ}^H_k(P^{\#}, T^{\#})$.

Construction of $P^{\#}$ and $T^{\#}$

Theorem [KNW25]

Given S and $\mathrm{MI}(x)$ for all $x \in S$, we can construct a grammar-like representation $P^{\#}$ and $T^{\#}$ of size $\widetilde{\mathcal{O}}(k)$ in time $\widetilde{\mathcal{O}}(k^2)$. The grammar supports $\widetilde{\mathcal{O}}(1)$ time `PILLAR` operations.

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- **[CKW20]**: output (representation of) $\mathrm{Occ}_k^H(P, T)$ using $\mathcal{O}(k^2)$ PILLAR operations.

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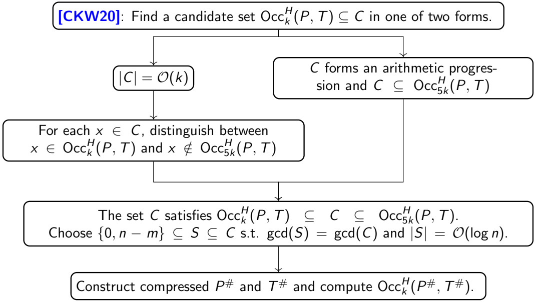
Theorem [KNW25]

Given N equality equations of the form $X[i..j) = X[i'..j')$ on a length-n string X, we can construct in time $\widetilde{\mathcal{O}}(N^2)$ a grammar-like representation of size $\widetilde{\mathcal{O}}(N)$ of a strings Y which:

1. satisfies all N equations, and
2. $Y[i] = Y[j]$ only when dictated by the equations.

Pattern Matching with Edits

The edit case has similar properties to the hamming case:



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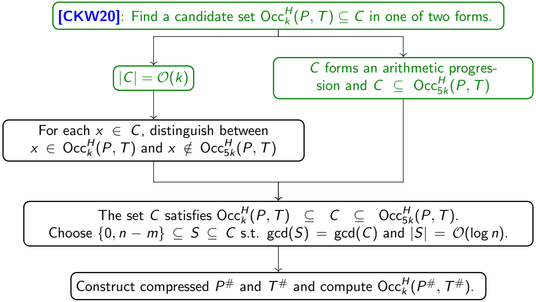

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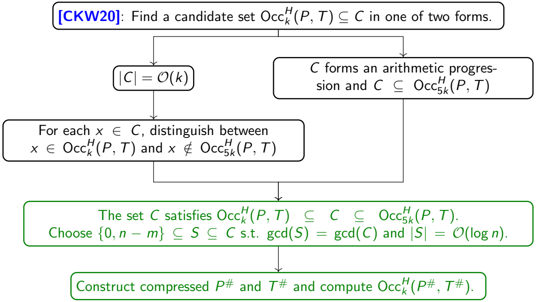

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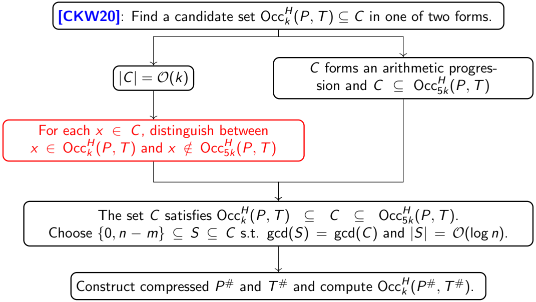
- **[KNW24]**: Adapt to the quantum setting the classical algorithm from **[GKKS22]** for the GAP EDIT DISTANCE problem, i.e., distinguish between $\mathrm{ED}(X, Y)$ "small" and $\mathrm{ED}(X, Y)$ "large".



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- An oracle $f : U \to \{0, 1\}$ is provided with the following properties:
  - $\Pr[f(i) = 1] \geq 9/10$ if $i \in I^+$,
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We give a $\widetilde{\mathcal{O}}(\sqrt{n})$-time algorithm, adapting the quantum algorithm for bounded-error inputs from **[HMDW03]**.

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Thank you!